Pre-Calculus CP 1 – Section 9.5 Notes **Name:** ______________________________

Pascal's Triangle & The Binomial Theorem

Blaise Pascal (1623-1662) formed a pattern of numbers to create an arithmetic triangle.

Each row starts and ends with a 1 but each entry is found by adding the two numbers from above.

Row 0	1
Row 1	1 1
Row 2	1 2 1
Row 3	1 3 3 1
Row 4	1 4 6 4 1
Row 5	1 5 10 10 5 1
Row 6	1 6 15 20 15 6 1

List the entries in Row 7.

The third entry in the row 5 is 10.

What is the fifth entry in row 6? ___________

The top row is called row zero and only contains one entry.

The next row down is called row 1 and contains two entries.

The next row down is called row 2 and contains three entries.

Therefore row n of Pascal's triangle contains ________ entries.

Each row exhibits symmetry.

For example, the third entry in from left must equal the third entry in from the right.

Sum all the entries in each row.

Pattern? The sum of all the entries in row n is equal to _______

Suppose you want to find the 11^{th} entry in row 34 of Pascal's Triangle. You certainly don't want to have to write out all the rows in order to get that entry.

SO WHAT SHOULD WE DO?!?

Here is another way of looking at Pascal's Triangle: using combinations! (Math→PRB)

Each row can be expressed as a combination:

$${}_0C_0$$
$${}_1C_0 \quad {}_1C_1$$
$${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$$
$${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$$

Row n starts with ${}_nC_0$ and continues through ${}_nC_n$ to yield n+1 entries.

The kth entry in row n of Pascal's Triangle is ${}_nC_{k-1}$.

Thus the 4th entry in row 12 of Pascal's Triangle is ${}_{12}C_3 = 220$.

To find ${}_{12}C_3$ on your calculator enter 12 then go to MATH→PRB and select number three and then enter 3 and hit enter to get an output of 220.

What is the 8th entry in row 13?

So why is all of this important and useful?

Given three expansions of the binomial (x+y):

$(x+y)^0 = 1$

$(x+y)^1 = x+y$

$(x+y)^2 = (x+y)(x+y) = x^2 + 2xy + y^2$

Now find $(x+y)^3$

Give your answer in standard form: the powers on the x in decreasing order. Also for each term in your expansion, put "x" before "y."

When expanding $(x+y)^n$ ...

- The coefficients can be found in ___________________
- The powers on the x variable start at _____ and go down to _____.
- The powers on the y variable start at ______ and go up to ______.
- In each term of the expansion, the powers on x and the powers on y sum up to __.

Putting all of these patterns together gives us the Binomial Formula!

Examples:

1) Expand the expression using the Binomial Formula:

$$\left(x+3\right)^5$$

2) Expand the expression using the Binomial Formula:

$$\left(4x-1\right)^3$$

3) Use the Binomial Formula to find the indicated coefficient or term:

The coefficient of x^2 in the expansion of $(2x-3)^9$

The sixth term of the expansion of $(3x+2)^8$